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Delineation of buildings at Machu Picchu by Inca builders – is metrological analysis of building outlines possible without the invasive survey?

Abstract

The metrological analysis of the Inca architectural elements at Machu Picchu using the cosine quantogram method was preceded by tests of different models of building delineation and erection, which aim to test the method's effectiveness in detecting the quantum with a differently defined error occurring in the survey sample. The sources of such errors may lie in the work of Inca builders and the modern researcher. The measurements of the architectural elements represent data from 3D laser scanning. The data obtained in this way refer to length measures that may have been delineated using quantum: building outlines, dimensions and distribution of doorways, windows and niches. However, it is uncertain which of them were actually planned by the Inca builders using the measure we are looking for (quantum) and which are incidental or outcome values.

Due to the sloping of the external walls towards the inside of the building, which was characteristic feature of Inca architecture, and the way in which the buildings were planned (on horizontal terrain, sloping terrain or with projection onto the ground), the level at which the building is available for measurement also has an important influence on the results of the analyses. This may be analogous to the level at which the building was planned, or it may be significantly above that level. As a result of these factors, the group of tests concerns theoretically predictable combinations of the ways in which the Inca buildings were delineated and the level available for measurement.

Tests of the effectiveness of the cosine quantogram method for Inca building outlines have shown that if they were laid out using some fixed measure (quantum) then, regardless of how the building was delineated: levelled terrace, sloping ground, projection from level to sloping ground, the cosine quantogram method must reveal its existence.

Key words: Peru, Inca architecture, building techniques, cosine quantogram, Cusco

Introduction

Research on Inca metrology with the example of analysing the architecture of Machu Picchu has been inspired by Professor Jacek Kościuk, who shared with students of Architecture his passion for studying historical architecture and the proportions encoded within it. In my case, this resulted in my doctoral thesis under the supervision of Professor Jacek Kościuk and Professor Mariusz Ziółkowski. Without their support and assistance at every stage of my research, it would have been impossible to find out whether the Incas used a standardised system of measurements in the construction of royal buildings.

Within the scope of the dissertation, a metrological analysis of the interior layout of Inca buildings, i.e., the niches and the spaces between them, was carried out using the cosine quantogram method, as well as an analysis of the outlines and divisions of the fronts of the buildings, as a result of which the use of two modules was observed: 42 cm and 54 cm. Even more interesting was the variation in the use of modules in terms of the purpose of the building: belonging to the Inca elite or their servants [1].

The presented work is a study of the potential of statistical analysis of building outlines, for which we have much less measurement data than the division of the walls of the interior of the buildings and the measurement of the structure is limited only to the contemporary ground level. Therefore, metrological studies based on the statistical method should be preceded by a technological analysis of the method of delineation of buildings

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situated on artificial terraces on the hillside. The system of terraces and the resulting plazas required considerable interference with the natural landscape and prior planning of the use of individual areas within a single establishment. The technology for constructing the terraces required engineering and construction skills, but also hydraulic skills that had already been developed by earlier Andean cultures. However, it was not until Inca times that projects of this type began to be realised on such a scale. The Cuzco region, and in particular the Urubamba river valley, has the most elaborate architecture of this kind. The Incan architecture built in this area was mainly located on terraces, less frequently on levelled grounds.

The planning and layout of the rebuilding of the capital of the Inca Empire at Cuzco, including the layout of the Temple of the Sun, was described in the mid-16th century by a Spanish chronicler [2]. The historical description of the transformation of the centre of power is attributed to Pachakuti Inqa Yupanka, who was said to have personally drawn the plans for the buildings and, with the help of imperial officials, supervised the construction. According to the description, the ruler used a measuring tool in the form of a rope (quechua: ñaña waska [3]). Its length is not known from chronicle descriptions, nor has material evidence of its existence been found. However, it can be surmised that if such a physical standard of measurement existed then its length was a multiple of a basic modulus – the derivative of one of the anthropometric measures.

Proving the existence of a unit of measurement or a system of measurement units that the Incas used should therefore take place on the basis of a metrological analysis of the constructions considered to be imperial projects. In the cosine quantogram method chosen for the purpose of such analysis, the existence of a specific unit of measure is not assumed, but it is verified whether, in the selected series of measurements, we can prove the existence of a basic unit of measure (module) – the quantum [4]–[7]. For Inca architecture, the cosine quantogram method has so far only been used at the urban scale. It has proven itself as a method in the analysis of cultivated terraces located in the Sacred Valley (Yukay-Urubamba Valley). According to a study by I. Farrington, the estimated quantum value for about 400 measurements is 1.615 m [8]. This corresponds to the value of the rikra, an Incan measure known from Spanish colonial-era chronicles and quechua dictionaries. A multiplication of this value is the waska (4 times the rikra), a measure used to measure greater distances with a rope. Here, we can expect the use of both familiar measures such as rikra and waska on the outlines of buildings, but also smaller units mentioned in historical sources, such as: half of the fathom (sikya), khococ (cubit), chaqui (foot) [9].

The architecture of the Machu Picchu site, the construction of which is historically identified with Pachakuti Inqa Yupanki, the same ruler who was said to have used a rope to measure buildings in Cuzco, seems to be appropriate research material for the search for a basic unit of measurement used as an architectural module.

Unlike other examples equally representative of metrological studies, such as the temple of the Sun in Cuzco itself (which, however, was largely a subject of reconstruction), Machu Picchu represents an architecturally homogeneous complex, constrained by the local topography. Here we find clear traces of conscious planning evident, for example, in the artificial landscaping in the form of terraces and levelled ceremonial plazas. The extent of the reconstruction of the architectural relics of Machu Picchu since its rediscovery by H. Bingham is also known [10].

Due to the fragmentary nature of the archaeological research of the foundation of buildings in the area, it is not possible to determine whether the priority for marking out the foundations of buildings was to prepare the terraces on which the foundation of the building was then laid out, or whether the building was laid out on an incline and the area around the building was later levelled to the appropriate height [11].

Testing quantum detection at different levels available for measurement

Due to the non-invasive nature of the survey, architectural measurements from the 3D point cloud were collected only above the current ground level, which in only a few cases will correspond to the level at which the building outline was measured by the former builders. Due to the design of the walls, which are inclined inwards at an angle of 3 to 6 degrees [12], [13], the results of the building outline measurements vary depending on the ground level available for measurement. The quantum detection tests presented below are intended to verify how different depths of the building arrangement affects the value of the quantum estimation. The basic assumption of this test is the existence of a level or slope of the ground on which the building plans were laid out according to a given unit of measurement.

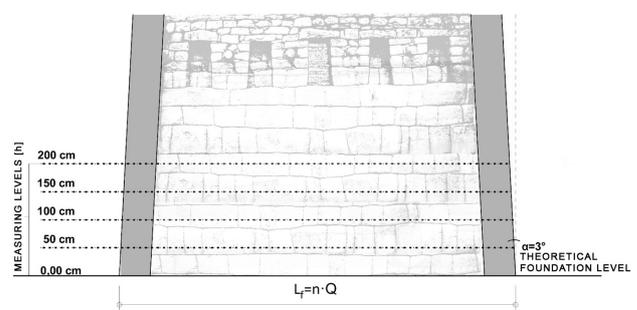


Fig. 1. Schematic of quantum analysis for the different levels available for measurement in relation to the theoretical foundation height on the levelled terrain (elaborated by A. Kubicka-Sowińska)

II. 1. Schemat analizy quantum dla różnych poziomów dostępnych do pomiaru w stosunku do teoretycznej wysokości rozmiernia obrysu na terenie zniwelowanym do poziomu (oprac. A. Kubicka-Sowińska)

For the testing models of the buildings, the already mentioned characteristic feature of Inca architecture was taken into account: the slope of the walls, which for the buildings of the Machu Picchu complex is between 3 and 4 degrees. Each of the tested models consists of 20 randomly generated dimensions, which are multiples of the quantum, set as 81 cm (value of measure related to *sikya*-half of a phantom known as *rikra*). The theoretical foundation level in the models tested is no more than 200 cm from the current ground level, the maximum value of which was determined on the basis of the foundation part of the Incan buildings studied so far, whose depth usually varies between 40 and 60 cm and sometimes reaches more than 100 cm [14]. Furthermore, the construction of the foundation walls was not made by the Incan builders under such a steep slope as the building walls and in some cases featured a foundation offset. If all these variables were taken into account for testing purposes, the number of combinations would increase exponentially. For this reason, a simplified model of the wall and foundation structure was adopted, taking into account the variables α (wall angle), β (site angle), h (foundation depth). The maximum value for the depth of foundations adopted for testing will also compensate for differences when there are offsets or a reduced angle of inclination of the foundation walls. In order to capture the variation in the estimated quantum value, each building arrangement was carried out at depths of: 50 cm, 100 cm, 150 cm, 200 cm, assuming a wall inclination of 3 degrees.

Layout of building plans on levelled ground

From the randomly generated multiples of the quantum situated at the layout level, the lengths of the building outlines were calculated in proportion to the slope of the walls and the height of the cut (Fig. 1). The trigonometric functions for the angle α (1) were used:

$$(1) \quad L_0 = nQ - 2(h \operatorname{tg} \alpha)$$

where:

L_0 – length of building outline at measurement level h

Q – quantum 81 cm

$n \in \mathbb{N}$

α – wall angle

The results of the quantum estimation (Fig. 2) are presented using randomly generated data (Table 1).

For this test model, a quantum of 81 cm is still detectable even at a measuring level 200 cm away from the theoretical foundation level. The quantum value and its score varies slightly from 79.9 cm (score: 6.80) to 81 cm (score: 7.8) as the distance from the foundation level increases. The result of the analysis shows that if there was a foundation level of the Incan buildings on the flat terrain then using the cosine quantumogram method applied to the building outline data accessible above foundation level will allow to find the quantum. Its value will be a deviation from the foundation level value, but it will still be a representation of the maximum of the function and statistically it will not be a significant difference.

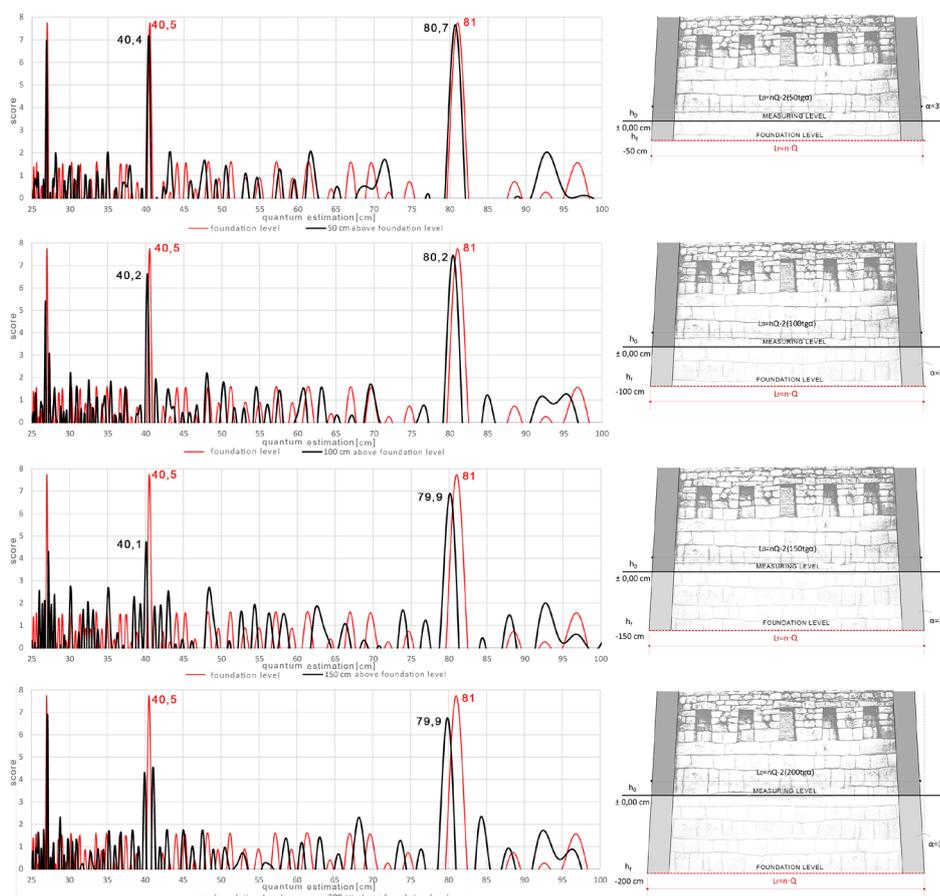


Fig. 2. Layout of building plans on levelled ground (elaborated by A. Kubicka-Sowińska)

II. 2. Rozmieranie planów budynków na terenie zniwelowanym do poziomu (oprac. A. Kubicka-Sowińska)

Table 1. Randomly generated data sample with a quantum multiple of 81cm for the test: Layout of building plans on levelled ground (elaborated by A. Kubicka-Sowińska)

Tabela 1. Losowo wygenerowana próba danych z wielokrotnością quantum 81cm do testu: Rozmierzanie planów budynków na terenie zniwelowanym do poziomu (oprac. A. Kubicka-Sowińska)

No.	Multiplication	Measurement of the building [cm]
1	12	972
2	13	1053
3	11	891
4	11	891
5	22	1782
6	7	567
7	20	1620
8	24	1944
9	24	1944
10	11	891
11	22	1782
12	12	972
13	19	1539
14	17	1377
15	5	405
16	13	1053
17	14	1134
18	9	729
19	15	1215
20	5	405

Layout of building plans on an sloping terrain

Another model assumes that the outline of the building was delineated on an inclined terrain (5°, 10°, 15°) (Fig. 3). This example was considered because of the irregular profile of the bedrock on which the entire Machu Picchu site was constructed. This also assumes that when the contour of the foundations was laid out, the ground was not levelled and the building was laid out on an inclination. This assumption can be justified by one of the characteristics of the Incan stonework technique, in which successive layers of stone blocks were adjusted to fit irregularities in the ground, such as outcrops of bedrock. The largest stone blocks were treated in a similar way and were left in their original position by adjusting successive layers of construction to them.

Randomly generated quantum multiples at the foundation level located on the sloping terrain were projected onto a horizontal cutting plane corresponding to the theoretical measurement level. Given the angular dependencies of the slope of the ground (β) and walls (α) and the levels available for measurement, 50, 100, 150, 200 cm below the foundation level. The building outline were calculated according to the following equation: (2).

$$(2) \quad L_0 = nQ (\cos \beta - \sin \beta \sin \alpha) - 2htg\alpha$$

Table 2. Randomly generated data sample with a quantum multiple of 81cm for the test: Delineating building plans on an inclined terrain: 5°, 10°, 15° (elaborated by A. Kubicka-Sowińska)

Tabela 2. Losowo wygenerowana próba danych z wielokrotnością quantum 81cm do testu: Rozmierzanie planów budynków na skłonie terenu: 5°, 10°, 15° (oprac. A. Kubicka-Sowińska)

No.	Multiplication	Measurement of the building [cm]
1	23	1863
2	22	1782
3	23	1863
4	19	1539
5	18	1458
6	21	1701
7	21	1701
8	13	1053
9	10	810
10	17	1377
11	8	648
12	11	891
13	16	1296
14	8	648
15	11	891
16	18	1458
17	13	1053
18	25	2025
19	23	1863
20	5	405

where:

L_0 – length of building outline at measurement level h
 Q – quantum 81 cm

$n \in N$

α – wall angle

β – the angle of inclination of the terrain on which the building is placed

The results of the quantum estimation (Figs. 4-6) are presented using randomly generated data (Table 2).

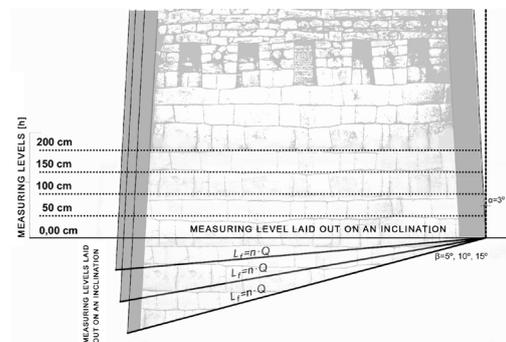


Fig. 3. Schematic of quantum analysis for the different levels available for measurement in relation to the theoretical foundation height on the inclination of the terrain (elaborated by A. Kubicka-Sowińska)

Il. 3. Schemat analizy quantum dla różnych poziomów dostępnych do pomiaru w odniesieniu do teoretycznej wysokości fundamentu utworzonego na nachyleniu terenu (oprac. A. Kubicka-Sowińska)

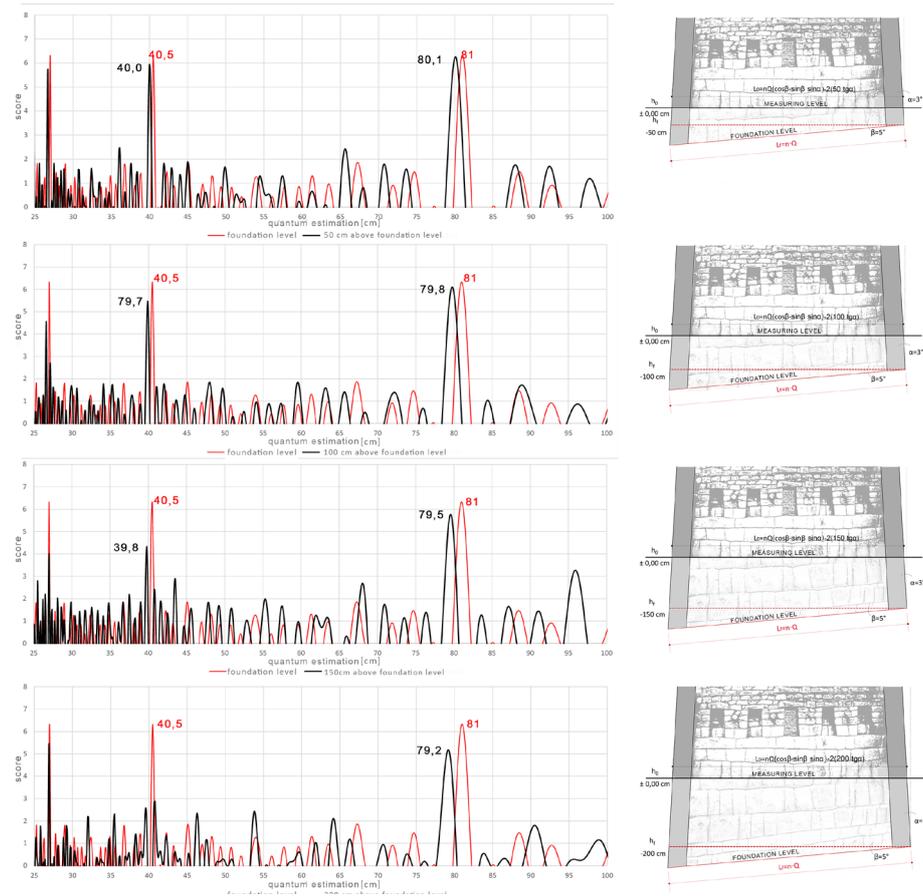


Fig. 4. Delineating the building plans on an inclined terrain of 5 degrees (elaborated by A. Kubicka-Sowińska)

II. 4. Rozmierzanie planów budynku na skłonie terenu równym 5 stopni (oprac. A. Kubicka-Sowińska)

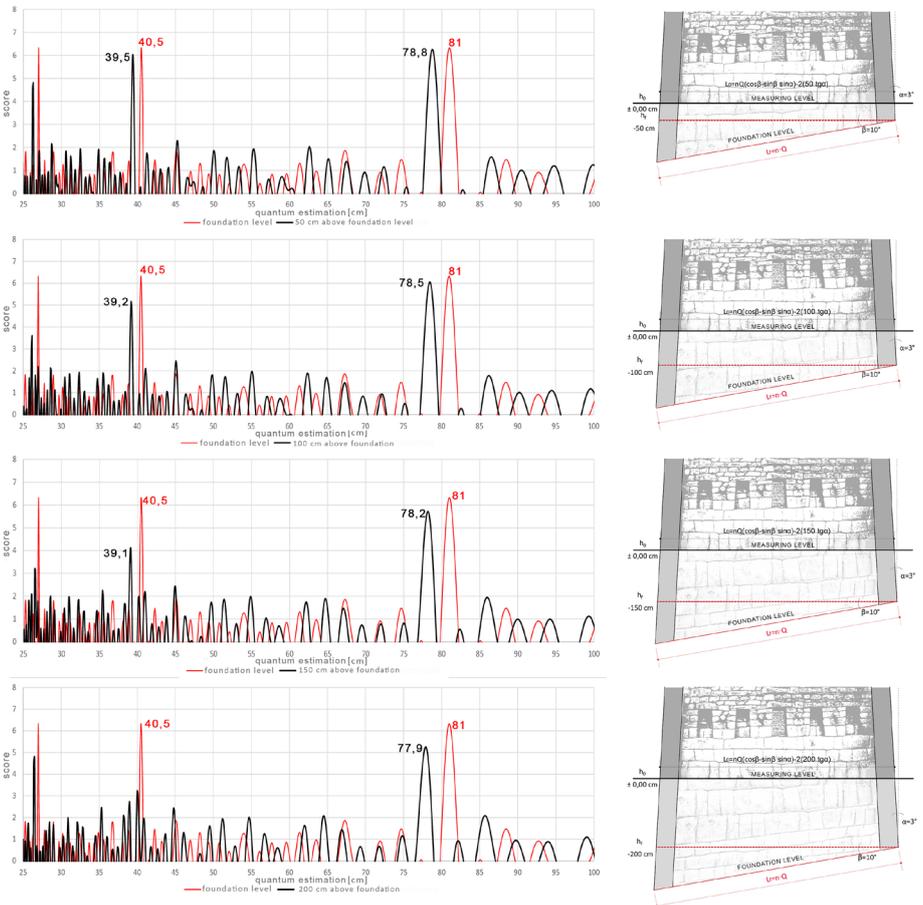


Fig. 5. Delineating the building plans on an inclined terrain of 10 degrees (elaborated by A. Kubicka-Sowińska)

II. 5. Rozmierzanie planów budynku na skłonie terenu równym 10 stopni (oprac. A. Kubicka-Sowińska)

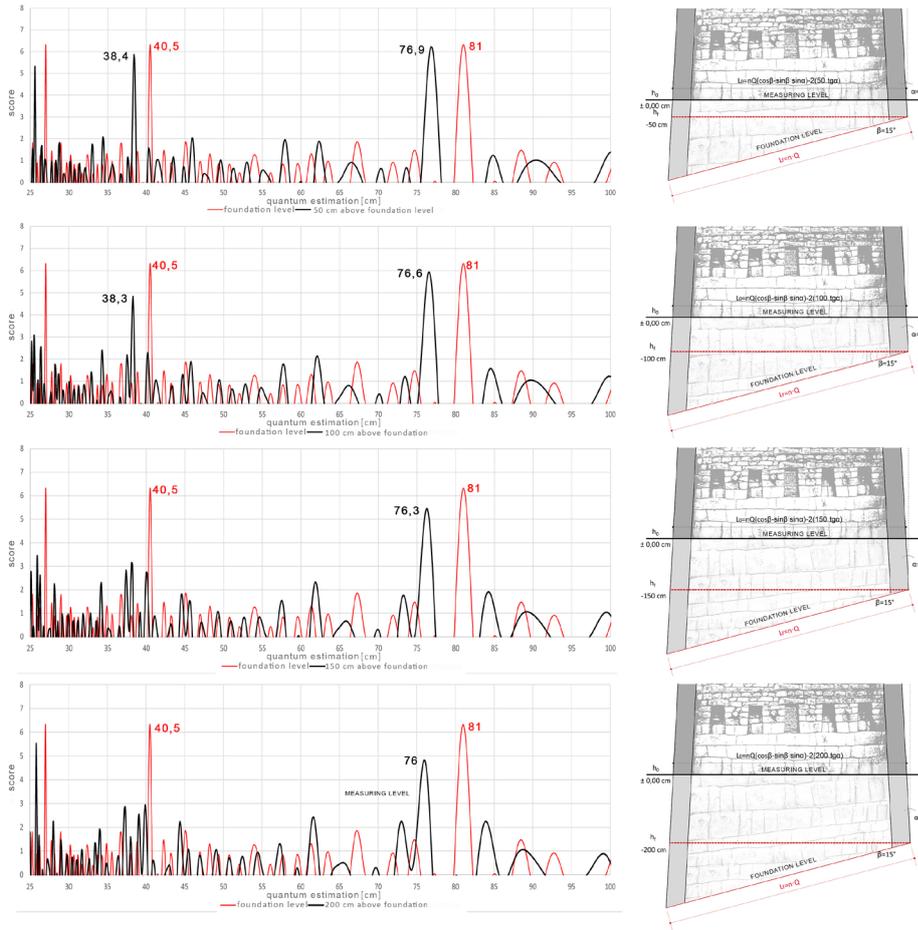


Fig. 6. Delineating the building plans on an inclined terrain of 15 degrees (elaborated by A. Kubicka-Sowińska)

II. 6. Rozmieranie planów budynku na sklonie terenu równym 15 stopni (oprac. A. Kubicka-Sowińska)

For randomly generated measurements with a multiple of 81 cm, delineated on a slope of 5 degrees, quantum is still detectable as a maximum function up to 200 cm below the level available for measurement. Its value and score change slightly: the value $80.1 \div 79.2$ cm and the score $6.32 \div 5.17$. Increasing the slope of the terrain to 10 degrees has the expected effect of decreasing the calculated quantum value $78.8 \div 77.9$ cm in relation to the assumed 81 cm. The score remains basically the same as in the previous test. For a slope of 15 degrees, the calculated quantum value is even lower $76.9 \div 75$ cm. At a distance of 200 cm, from the foundation level, concurrent quantum values appear (25.8; 61.7; 73.1). Confidence interval calculated for the entire quantum estimate, reduce the reliability of the obtained result. As in the previous tests taking into account the error occurring in the measured quantum and here in each quantum estimation variant the sub-values: 1/2; 1/3; 1/5 of the quantum, significantly reduce their score.

Layout of building plans horizontally with projection of multiples of quantum on the inclined terrain

Laying foundations on sloping ground can also imply another method of delineation – the projection of horizontally measured distances onto sloping terrain. This case further increases the difference between the dimensions of the buildings at the level of foundation and the length buildings collected at the level available for measurement.

The test was carried out for values of wall angle ($\alpha=3^\circ$) and slope of the terrain ($\beta=5^\circ, 10^\circ, 15^\circ$) (Fig. 7).

For randomly generated data containing multiples of quantum 81 cm, the lengths of the building outlines (L_0) were calculated taking into account the heights h of the inclination angles α and β according to formula (3).

$$(3) \quad L_0 = nQ \operatorname{tg} \alpha \left(\operatorname{ctg} \alpha - \frac{2h}{nQ} \right) - \operatorname{tg} \beta$$

where:

L_0 – length of building outline at measurement level h
 Q – quantum 81 cm

$n \in N$

α – wall angle

β – the angle of inclination of the terrain on which the building is placed

The results of the quantum estimation (Fig. 8) are presented using randomly generated data (Table 3).

Despite the different way of delineation of the building, the results of the quantum estimation for a slope of 5 degrees, at different measurement levels are very similar to the previous test. The value of the obtained quantum with the increase of the depth changes from

Table 3. Randomly generated data sample with a quantum multiple of 81cm for the test: Horizontal building plan layout with projection of multiples of quantum onto slope: 5°, 10°, 15° (elaborated by A. Kubicka-Sowińska)

Tabela 3. Losowo wygenerowana próba danych z wielokrotnością quantum 81cm do testu: Rozmierzanie planów budynków w poziomie z odrzutowaniem wielokrotności quantum na skłon terenu: 5°, 10°, 15° (oprac. A. Kubicka-Sowińska)

No.	Multiplication	Measurement of the building [cm]
1	19	1539
2	23	1863
3	17	1377,0
4	6	486
5	24	1944
6	24	1944
7	18	1458
8	25	2025
9	11	891
10	21	1701
11	7	567
12	20	1620
13	23	1863
14	20	1620
15	16	1296
16	25	2025
17	12	972
18	21	1701
19	19	1539
20	23	1863

Table 4. Results for the test: Horizontal building plan layout with projecting multiples of quantum per slope: 10°, 15° (generated by A. Kubicka-Sowińska)

Tabela 4. Wyniki dla testu: Rozmierzanie planów budynków w poziomie z odrzutowaniem wielokrotności quantum na skłon terenu: 10°, 15° (oprac. A. Kubicka-Sowińska)

Inclination of the terrain	Measuring level [cm]	Maxima of the function (quantum) [cm] and score
10°	50	79,9 ; 6,2
	100	79,5 ; 6
	150	79,1 ; 5,6
	200	26,9 ; 5,39
15°	50	79,5 ; 6,4
	100	79,1 ; 6,1
	150	78,8 ; 5,6
	200	78,4 ; 5

80.2÷79.2 cm in relation to the assumed 81 cm, and its score decreases from 6.2÷5.2. Due to the similarity of the results, the graphical representation of the result of this test has been limited only to the ground inclination angle $\beta=5^\circ$, and the remaining results for the angle $\beta=10^\circ$ and $\beta=15^\circ$ have been presented in the form of

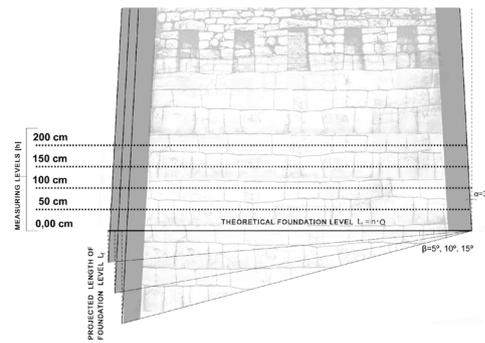


Fig. 7. Schematic of quantum analysis for different measuring level with the foundation level delineated horizontally and then projected the length on the slope of the terrain (elaborated by A. Kubicka-Sowińska)

II. 7. Schemat analizy quantum dla różnych poziomów pomiaru planu budynku z uwzględnieniem rozmierzania quantum w poziomie i odrzutowaniem odległości na skłon terenu (oprac. A. Kubicka-Sowińska)

a table (Table 4). Similarly to the previous test, the value of the quantum obtained changes with increasing slope and reaches similar values.

Summary of tests of the effectiveness of the cosine quantogram method in detecting quantum in the outlines of Inca buildings

Tests of the effectiveness of the cosine quantogram method for the outlines of Incan buildings have shown that if they were delineated using some fixed unit of measurement (quantum) then, regardless of the way in which the building was delineated (levelled terrace level, slope of the terrain, projection from level to slope of the terrain), the cosine quantogram method must reveal its existence. Based on the assumption that the building outlines were laid out using the basic unit of measurement, we can prove its existence up to a distance of 200 cm between the hypothetical foundation level and the level available for measurement. In the tests presented above, the sloping wall construction and the slopes of the terrain obviously affected the value of the quantum itself. In the most extreme case, the difference in quantum value was 5 cm. This leads to the conclusion that in the case of long dimensions with numerous multiples of the quantum (e.g., building outlines), we are not able to determine unambiguously what the exact value of the standard measure was, but we can certainly prove its existence. Quantum analysis of buildings outline which were laid out using a module will appear in the function graph as a maximum point but its value could be interrupted by the distance to original foundation level.

The observations from this experiment were taken into account during the metrological analysis of the building outlines at Machu Picchu. Wide range of bootstrap confidence intervals calculated for the quantum results shows

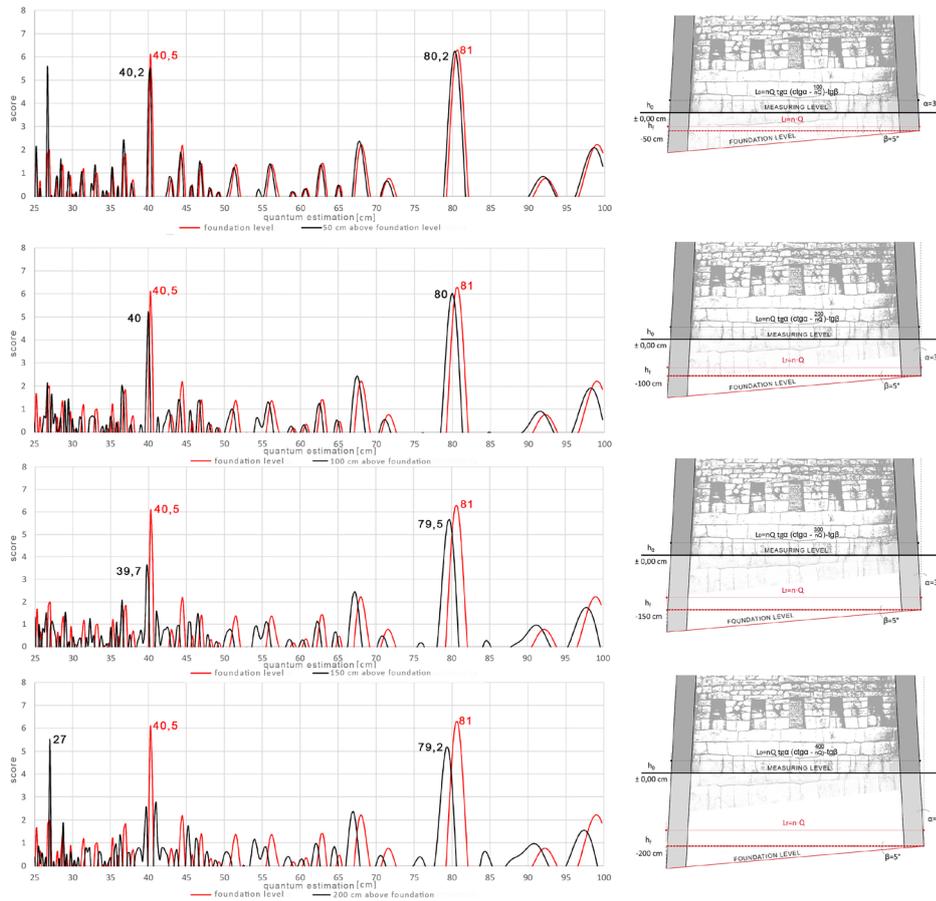


Fig. 8. Delineation building plans horizontally with a projection of multiples of quantum on a slope of 5 degrees (generated by A. Kubicka-Sowińska)

II. 8. Rozmieranie planów budynków w poziomie z drutowaniem wielokrotności quantum na skłon terenu równy 5 stopni (oprac. A. Kubicka-Sowińska)

that in the outlines of Machu Picchu houses there is no delineation using quantum (module). At least there is no statistical evidence of using a quantum in building outlines by ancient construction workers. A negative result from the use of the module may be due to both too few measurements (less than 20) and the inclusion in the measurement sample of a large number of dimensions that were not measured using the module, but are, for example, the result of the available tract depth (depth of terrace). For example, only one wall – the front one – could be determined in the building outline, while the length of the others was a result of the size of the available terrain. It should be noted, however, that with the terrain being so difficult to manage, most of the main dimensions of the building were rather dictated by the topography of the site.

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Streszczenie

Wytyczanie budynków w Machu Picchu przez inkaskich budowniczych – czy analiza metrologiczna obrysów budynków jest możliwa bez inwazyjnych badań?

Analizę metrologiczną elementów architektury inkaskiej na Machu Picchu przy użyciu metody cosine quantogram poprzedziły testy różnych modeli wytyczania i wznoszenia budynku, które mają na celu sprawdzenie skuteczności metody w wykrywaniu quantum przy różnie zdefiniowanym błędzie występującym w badanej próbie pomiarowej. Źródła takich błędów mogą leżeć zarówno po stronie inkaskich budowniczych, jak i współczesnego badacza. Pomiar elementów architektury stanowią dane zebrane podczas laserowego skanowania 3D. Dane pozyskane w ten sposób odnoszą się do miar długości, które mogły być rozmiarowane z użyciem quantum: obrysów budynków, wymiarów pomieszczeń, wymiarów i rozmieszczenia otworów drzwiowych, okiennych i nisz. Nie ma jednak pewności, które z nich były faktycznie rozmiarowane przez inkaskich budowniczych z użyciem poszukiwanej przez nas miary (quantum), a które są wartościami przypadkowymi lub wynikowymi.

Ze względu na charakterystyczne dla architektury inkaskiej pochylenie ścian zewnętrznych do wnętrza budynku oraz nierozpoznany do tej pory sposób rozmiarowania budynków (na poziomym terenie, skłonie terenu lub z odrzutowaniem na teren), istotny wpływ na wyniki analiz ma też poziom, na którym budynek jest dostępny do pomiaru. Może on być analogiczny do poziomu, na którym rozmiarowano budynek lub też być położony znacznie powyżej tego poziomu. W związku z tymi czynnikami poniższa grupa testów dotyczy dających się teoretycznie przewidzieć kombinacji sposobów rozmiarowania budynków inkaskich i poziomu dostępnego do pomiaru.

Testy skuteczności metody cosine quantogram dla obrysów budynków inkaskich wykazały, że jeśli były one rozmiarowane z użyciem jakiejś stałej miary (quantum) to niezależnie od sposobu wytyczania budynku (zniełowany poziom tarasu, skłon terenu, odrzutowanie z poziomu na skłon terenu) to metoda cosine quantogram musi ujawnić jej istnienie.

Słowa kluczowe: Peru, architektura inkaska, techniki budowy, cosine quantogram, Cuzco