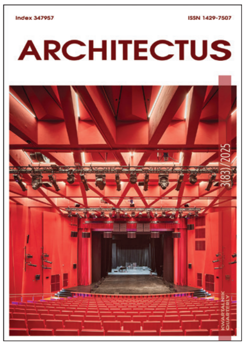

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On the variability in time of the longitudinal modulus of elasticity E and the traverse modulus of elasticity G and their impact on the rigidity of timber structures

Abstract

The paper formulated the problem of lack of analysis of changes in rigidity over time, large-space constructions of glued wood, describing the dome of Konohana Dome in Japan. On the example of a study of pine beams with a static support scheme, it was shown that on the basis of the bending measurement it was possible to estimate the spring and form rigidity of the research model. It has been shown that with the increase in bending over time, the moduli of longitudinal E and transverse G elasticity decreases. It has been revealed that the cause of the nonlinear increase in bending over time is the decreasing rigidity of the spring and the shape of the support beam.

Key words: modulus of longitudinal rigidity, modulus of transverse rigidity, critical force, structural rigidity

Introduction

The building industry of glued wood has developed dynamically since the forties of the 20th century, since the development and implementation on industrial scale of water-proof resorcinol glues on the basis of resins (RPF). Since the seventies of the 20th century large-capacity facilities having the function of sports and spectators halls have been built. One of the examples is the little-known in Europe the multi-function spectators hall, Konohana Dome, at Miyazaki in Japan. In the Polish and global references no analysis in time of the rigidity of the construction of such facilities has been found. Referring to the study by Zbigniew Kowal (1996) and using the analogy of the structure of wood to the building of fibrous composites, the author of this paper has conducted her own experiment, revealing that on the basis of the measurement of deflections it is possible to assess the tendency of variations in time of the rigidity of wooden constructional elements and systems.

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Description of the research problem

The large-spatial dome, Konohana Dome (Fig. 1), was built in 2004 of glued wood (Iimura 2008). The facility constitutes a big achievement in the field of architecture, building technology and construction. Bars of glued wood were used therein, including thickset bars of a slimness of $\lambda < 20$, connected with steel knots. The steel knots were connected with wooden bars using steel sheets glued into the wooden section. Despite noticing a range of negative processes in such constructions, e.g., the increase in deflections of wooden elements in time without a rise in the load, appearing, among others, in the dome at Portland, USA (Misztal 2018), and the fission of the fibres of wooden bars in the zones at by steel nodes (Lachiewicz-Złotowska, Orłowicz 2008), no recognition research is conducted within the field of a change in the rigidity of construction wooden systems study in time.

The shape of the Konohana Dome was built with wood glued in layers from December 2002 until March 2004. The bars of the cantilever structure of a section 30 × 120 cm was made of the quickly-growing tree: Japanese cryptomeria (Sekkan-sugi, *Cryptomeria japonica*), appearing in the vicinity of the city of Miyazaki, growing for at least 40 years

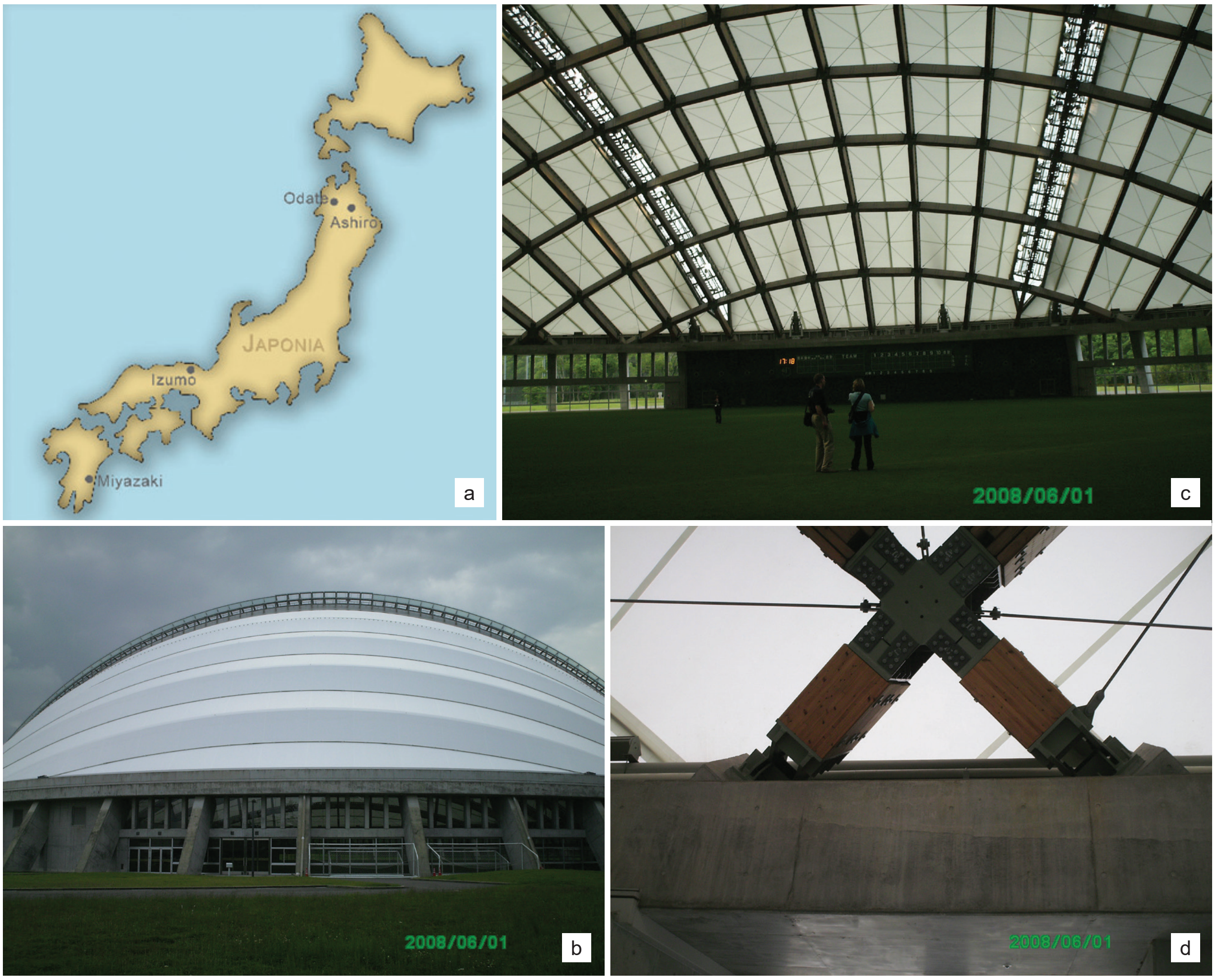

Fig. 1. Beam scaffolding, steel-wood Konohana Dome built with sugi wood in Miyazaki, Japan, 2004:
a) location of the Konohana Dome, b) view of the dome from the entrance side, c) view of the wooden structure from the inside,
d) view of the steel knot and constricted wooden bars in the support zone of the dome structure (photo by B. Misztal)

Il. 1. Ruszt belkowy, stalowo-drewniany kopuły Konohana Dome wybudowanej z drewna sugi w Miyazaki, Japonia, 2004:
a) lokalizacja kopuły, b) widok kopuły od strony wejścia, c) widok konstrukcji drewnianej od wewnątrz,
d) widok stalowego węzła i krępych prętów z drewna w strefie podporowej konstrukcji kopuły (fot. B. Misztal)

since after that time the sugi wood gets the strength sufficient to make the construction.

The dome having an elliptic shape of the horizontal projection has a diameter inside the projection of 122.0 × 104.5 m and the greatest height of 38.0 m. The total area of the projections amounts to 10,966.0 m^2. The volume of the glued wood, of which the wooden structure was made, amounts to 1,381.0 m^3. The mass of the wooden structure as converted into 1 m^2 of the projection, totals 42.80 kg/m^2, whereas the mass of the steel, of which the knots were made (Figs. 1c, d) and the steel structure that tightens the fabric constituting the outside shell totals 44.70 kg/m^2 (Iimura 2008).

The construction of the dome is a steel-wood beam scaffolding having a double curvature. Such a system is an arrangement of wooden bars susceptible to the action of transverse forces in the sections by the steel knots, especially in thickset bars of a small slimness (Fig. 1d). In the paper, *Strength Performance of Glulam Made of Obi-sugi Laminae with Low Young's Modulus of Bending* (Matsumoto et al. 2007), the method of determining the modulus of elasticity E of sugi wood, of which the structure of the dome at Miyazaki was described, however, the modulus of transverse elasticity G was not determined. The variations in time of the moduli of longitudinal elasticity E and that of transverse elasticity G was not examined.

The domes made of glued wood, including the dome at Miyazaki, are made of the natural fibrous composite, as wood is. The fibrous composites behave, unlike homogenous materials, such as steel. Kowal (1996) introduced a model of the theoretical analysis of constricted composite bars. When examining them, he found that the shape of the bar destruction depended on the bar slimness, the properties of fibres and the matrix as described by the moduli of longitudinal elasticity E and transverse elasticity G as well as on the value of the quotient E/G. Kowal discovered the paradox of the load carrying-capacity of composite bars: thickset composite bars of the slimness of $\lambda < 20$ show a lower critical load capacity than those of a higher slimness, e.g., $\lambda > 80$. The graphs compiled by the author of the paper (Misztal 2018) on the basis of the results of the study by

Kowal presents how grows the coefficient decreasing the load capacity η for bars of a slimness of $\lambda < 20$ (Fig. 2). The coefficient η is important since it tells us, how many times the critical load-bearing capacity of a constricted bar decreases depending on the slimness λ and on the relation E/G according to the equation by Kowal.

Since wood is a natural fibrous composite, similar phenomena should be expected as in artificial fibrous composites. For these reasons the author of the paper has been dealing, for many years, with the exploration of wood as a fibrous composite, including the moduli of elasticity of wood.

In the referenced literature of this subject matter no research by other authors has been noticed about a change in time of the moduli of elasticity E and G of wood, as well as on the critical load-bearing capacity of wooden structures as well as the variations of their rigidity in time.

Considering the impact of wood (as reflected by the mag nitudes of E and G) on the aesthetics, durability and safety of the facilities made of wood, the author of the paper conducted her own experiment in order to show how changes the value of the moduli of elasticity of wood in time. The information on the moduli of elasticity, both longitudinal E and transverse ones G, has the basic meaning for the development of the large-spatial facilities made of glued wood, the assessment of the rigidity and the load-bearing capacity in time. The knowledge of the magnitudes of E and G provides a possibility to forecast in time the variations of an architectonic form and the load-bearing capacity of a structure.

The objective of the research

The objective of the research conducted by the author of this paper is to show that:

1. The tendency of the variations of the moduli of transverse rigidity G and that of longitudinal elasticity E can be diagnosed on the basis of the measurements of deflections.
2. The rigidity of a wooden element changes in time.

Description of own research

Theoretical basics of the method

The author of the paper has dealt for many years with the theoretical basics of the research about the moduli of longitudinal elasticity and that of transverse elasticity. In her publication dated 2020 (Misztal 2020), she specified the algorithm to determine the values of longitudinal elasticity E and that of transverse elasticity G, as well as she examined the model of a spruce wood beam. Reminded in this paper is the course of calculations of the moduli E and G, in order to prove that the rigidity of a wooden element varies in time. To this aim, the author of the paper conducted an experiment on two models of a cantilever beam made of pine wood.

In the research scheme adopted for the research the total deflection of the supporting beam depends on the bending rigidity EJ and on the shape rigidity FG of the bar (E – modulus of longitudinal elasticity, G – modulus of transverse elasticity, J – moment of inertia of the bar section, F – cross-sectional area of the bar).

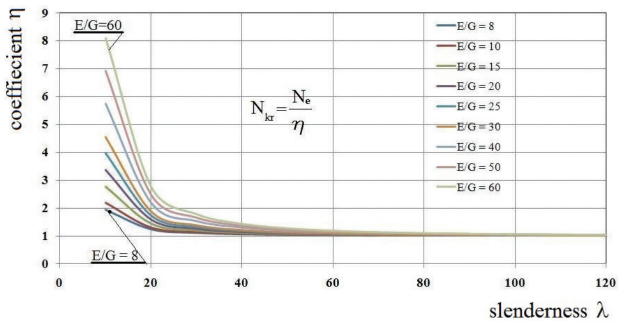

Fig. 2. Graph of the reduction factor η for the critical load-bearing capacity of compressed members according to equation (1) (elaborated by Misztal 2018)

equation (1): $N_{kr} = \dfrac{N_e}{\eta}$

N_{kr} – critical load of the composite member, N_e – Euler's critical load, η – reduction factor for the Euler-based critical load according to classical mechanics (source: Kowal 1996)

Il. 2. Wykres współczynnika η zmniejszającego nośność krytyczną prętów ściskanych według wzoru (1) (oprac. Misztal 2018)

wzór (1): $N_{kr} = \dfrac{N_e}{\eta}$

N_{kr} – siła krytyczna pręta kompozytowego,
N_e – siła krytyczna według Eulera,
η – współczynnik zmniejszający siłę krytyczną liczoną z wzoru Eulera według zasad mechaniki klasycznej (źródło: Kowal 1996)

An experiment lasting for 72 hours was scheduled. The examination of two cantilever beams having the length of $l_1 = 2l_2$ and l_2 was conducted (2). The system of equations (3) and (4) was arranged:

For the cantilever beam of the lenght $l_1 = 800$ mm, where $l_1 = 2l_2$ (2)

$$y_1 = \frac{Pl_1^3}{3EJ} + \frac{Pl_1}{FG} = \frac{8Pl_2^3}{3EJ} + \frac{2Pl_2}{FG} \quad (3)$$

For the cantilever beam of the lenght $l_2 = 400$ mm:

$$y_2 = \frac{Pl_2^3}{3EJ} + \frac{Pl_2}{GF} \quad (4)$$

Cantilever beam's cross-section:

$$F = 3.8 \times 1.03 \times 10^{-4}\ \mathrm{m}^2 = 3.914 \times 10^{-4}\ \mathrm{m}^2$$

Geometric moment of inertia of the cantilever beam's cross-section:

$$J = \frac{3.8 \times 10^{-2}\ [\mathrm{m}] \times (1.03 \times 10^{-2}\ [\mathrm{m}])^3}{12} =$$

$$= 0.3460302 \times 10^{-8}\ \mathrm{m}^4$$

The coefficients designated as A were introduced – equation (5) and B – equation (6) (5)

$$A = \frac{Pl_2^3}{3J} = \frac{2.50N \times (40 \times 10^{-2})^3\ \mathrm{m}^3}{3 \times 0.3460302 \times 10^{-8}\ \mathrm{m}^4} =$$

$$= 15412912.9 \left[\frac{\mathrm{N}}{\mathrm{m}}\right]$$

$$B = \frac{Pl_2}{F} = \frac{2.50N \times 40 \times 10^{-2}\ \mathrm{m}}{3.914 \times 10^{-4}\ \mathrm{m}^2} = 2554.9 \left[\frac{\mathrm{N}}{\mathrm{m}}\right] \quad (6)$$

The solution for the system of equation (3) and (4) are equations (7) and (8)

$$E = \frac{6A}{y_1 - 2y_2}, \quad A \text{ – equation (5)} \quad (7)$$

$$G = \frac{B}{y_2 - \frac{A}{E}}, \quad A \text{ – equation (5) and } B \text{ – equation (6)} \quad (8)$$

The formulae for the accounting determination of the values of the modulus of longitudinal elasticity E and that of transverse elasticity G using the measured deflections of the model of the cantilever beam were obtained.

Research

Selected for the research were beams made of pine wood in an air-dry state. The experiment was conducted in a room of a temperature of 22°C and a humidity of ca. 40%. The measurements of deflections were performed on the free end of the cantilever using an inductive sensor connected to a computer on which the values of deflections were recorded:

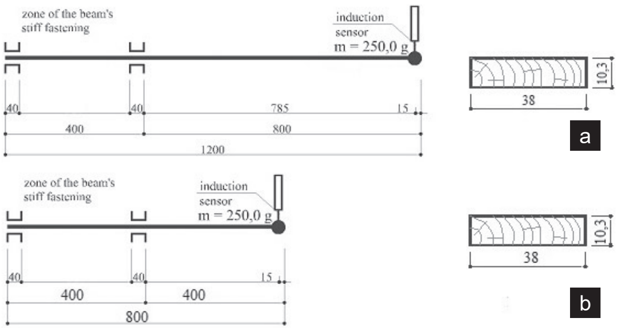

Fig. 3. Model of the tested beam with cantilever length: a) l_1 = 800 mm, b) l_2 = 400 mm (schematic for dynamic testing and cross-section) (elaborated by B. Misztal)

Il. 3. Model testowanej belki o długości wspornika: a) l_1 = 800 mm, b) l_2 = 400 mm (schemat belki do badań dynamicznych i przekrój) (oprac. B. Misztal)

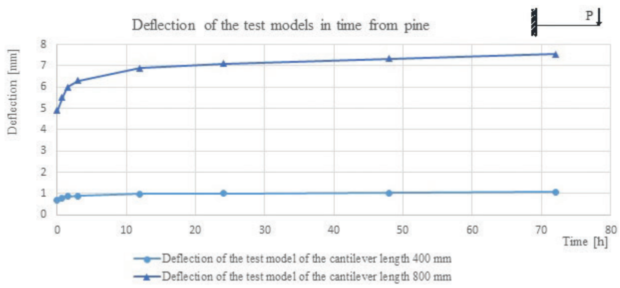

Fig. 4. Time-dependent deflection of the free ends of pine wood cantilevers (Fig. 3) during a 72-hour test (elaborated by B. Misztal)

Il. 4. Zmiana w czasie ugięć swobodnych końców wsporników z drewna sosnowego (Il. 3) w czasie próby trwającej 72 godzin (oprac. B. Misztal)

immediately upon the experiment for the time t_0 = 0.00, after the time t_1 = 0.75 hours, t_2 = 1.50 hours, t_3 = 3.00 hours, t_4 = 12.00 hours, t_5 = 24.00 hours, t_6 = 48.00 hours, t_7 = 72.00 hours.

The first research model had the following dimensions: the cross-section of 1.03 × 38 mm, the length of 1,200 mm, the second research model: cross-section 1.03 × 38 mm, length 800 mm. The models were weighed before the experiment. Considering a tiny mass, the own mass of the models was omitted. Fastened on the first research model, on one end, was the model of the cantilever beam according to the schematic shown in Figure 3a, having the length of the cantilver of 800 mm.

The load of the value of m = 250 g was applied on the free end of the cantilever perpendicularly to the plane of the lower rigidity of the beam. The temporary deflection was measured immediately, which amounted to y_1 = 4.91 mm = 4.91 × 10^{-3} m. After the time t_2 = 0.75 hours of the impact of the deflection was read out: it amounted to y_2 = 5.50 mm = 5.50 × 10^{-3} m. The successive deflections were measured over the time of 72 hours of the experiment's duration and were listed in Table 1. The graph of the deflections measured was shown in Figure 4.

Prepared for testing on the second test stand was another model having the static schematic, as shown in 4a, of the cantilever length of l_2 = 400 mm. The beam was fastened on one end, whereas the another, free end was loaded with a concentrated mass m = 250 g perpendicularly to the plane of the lower rigidity of the beam, as shown on the schematic in Figure 3b.

Immediately upon suspending the mass m = 250 g, the temporary deflection was measured in time t_0 = 0.00, the temporary deflection measured amounted to y_0 = 0.71 mm = 0.71 × 10^{-3} m. After the time t_1 = 0.75 hours, the impact of the load was re-read, the impact of the deflection amounted to y_1 = 0.79 mm = 0.79 × 10^{-3} m. Successive deflections were read out in the time of 72 hours and listed in Table 1. The graph of the deflections measured in function of time is shown in Fig. 4.

Results of the testing

Listed in Table 1 are the deflections measured during 72 hours of the experiment's duration.

The total increase in deflections of the model a cantilever length of $l_1 = 2l_2$ = 800 mm during 72 hours amounted to 2.64 mm, whereas that of the model of the cantilever length l_2 = 400 mm amounted to 0.36 mm. The increases in the deflections decreases in time, however, no stabilization of deflections during 72 hours of the experiment's duration were noticed.

While substituting the magnitudes of the deflections on the free ends of the cantilevers as listed in Table 1 and as shown in Figure 5 to the formulae (8) and (9), calculated were the moduli E (formula 7) and formula G (formula 8) of the pine-wood beam tested in the dry-air state. Listed in Table 2 are the calculated values of the moduli, whereas shown in Figure 5 is the graph of the decreasing of the values of the modulus of elasticity E and that of the modulus of transverse rigidity in G in time.

Table 1. Chart of deflections in function of the time measured during 72 hours of the experiment (elaborated by B. Misztal)
Tabela 1. Wartości ugięć w funkcji czasu zmierzonego w ciągu 72 godzin doświadczenia (oprac. B. Misztal)

Time	Model of the cantilever length 800 mm		Model of the cantilever length 400 mm	
	time [h]	deflection [mm]	time [h]	deflection [mm]
t_0	0.00	4.91	0	0.71
t_1	0.75	5.50	0.75	0.79
t_2	1.5	6.00	1.5	0.86
t_3	3	6.30	3	0.90
t_4	12	6.90	12	0.98
t_5	24	7.10	24	1.01
t_6	48	7.33	48	1.04
t_7	72	7.55	72	1.07

Table 2. List of modules of longitudinal rigidity and transverse rigidity *G* of tested beams (elaborated by B. Misztal)
Tabela 2. Zestawienie modułów sztywności podłużnej i sztywności poprzecznej *G* badanych belek wspornikowych (oprac. B. Misztal)

Time	Time [h]	Longitudinal modulus of elasticity *E* [GPa]	Transverse modulus of elasticity *G* [MPa]
t_0	0	26.49	19.91
t_1	0.75	23.59	18.69
t_2	1.5	21.60	17.42
t_3	3	20.55	17.03
t_4	12	18.79	16.31
t_5	24	18.20	15.64
t_6	48	17.61	15.48
t_7	72	17.09	15.18

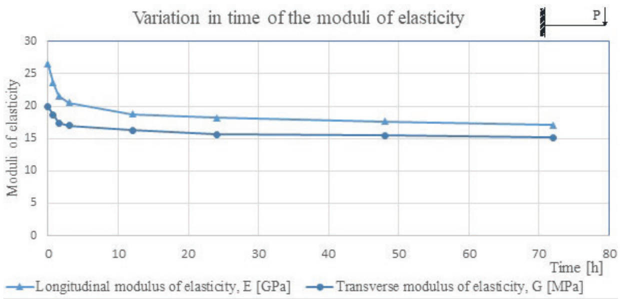

Fig. 5. Time-dependent changes in the modulus of longitudinal elasticity *E* and modulus of transverse rigidity *G* of pine wood cantilevers (Fig. 3) during a 72-hour test (elaborated by B. Misztal)

Il. 5. Zmiana w czasie modułów sprężystości podłużnej *E* i sztywności poprzecznej *G* wsporników z drewna sosnowego (il. 3) w czasie próby trwającej 72 godzin (oprac. B. Misztal)

Summary

The knowledge of the magnitudes of the longitudinal elasticity *E* and that of the transverse elasticity *G* as well as that of their variations in time has the meaning for the further development of wooden constructions, including glued wood. It allows to assess the load-carrying capacity of a construction and the deformation of architectonic forms in time. In the experiment conducted, the deflections of the loaded cantilevers increased in time and no stabilization of the deflections after 72 hours of the experiment's duration was noticed. It was demonstrated in this paper that on the basis of the measurements of deflections it was possible to assess the magnitudes of the moduli of longitudinal elasticity *E* and that of transverse elasticity *G*, as well as that those values decrease jointly with the rise in deflections in time.

The cause for the non-linear increase in deflections in time is the decreasing rigid elasticity *EJ* and shape rigidity *FG* of a beam made of pine-wood (*J* – moment of cross-sectional inertia, *F* – cross-section of the beam). The fact of decreasing the rigidity of wooden elements in time requires a further recognition, especially for large-spatial facilities made of glued wood.

The EC5 Standard considers the variability of deflections and the strength of wood in time through the coefficient k_{mod}, depending on the class of usage and that of last duration. However, both testing as well as the recognition of the variability of moduli and their strength in time shall be further deepened since they also depend on the features of the wood being a natural fibrous composite. The issue is important, especially in the case of introducing on the market of artificial composites using wood.

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Wojciech Mühleisen

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Streszczenie

O zmienności w czasie modułów sprężystości podłużnej E i poprzecznej G oraz ich wpływie na sztywność konstrukcji z drewna

W artykule sformułowano problem braku analizy zmian sztywności w czasie wielkoprzestrzennych konstrukcji z drewna klejonego, opisując kopułę Konohana Dome w Japonii. Na przykładzie badań belki z drewna sosnowego o schemacie statycznym wspornika wykazano, że pomiar ugięć pozwala oszacować sztywność sprężystą i postaciową modelu badawczego. Udowodniono także, że wraz ze wzrostem ugięć w czasie zmniejszają się moduły sprężystości podłużnej *E* i poprzecznej *G*. Ujawniono, że przyczyną nieliniowego przyrostu ugięć w czasie jest malejąca sztywność sprężysta i postaciowa belki wspornikowej.

Słowa kluczowe: moduł sztywności podłużnej, moduł sztywności poprzecznej, siła krytyczna, sztywność konstrukcji